UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Ordinary Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3804714867

ADDITIONAL MATHEMATICS **4037/21**

Paper 2 **October/November 2011**

2 hours

Candidates answer on the Question Paper.

No Additional Materials are required.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.

For Examiner's Use	
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This document consists of **18** printed pages and **2** blank pages.

DC (CW/CGW) 35877/3

UNIVERSITY *of* CAMBRIDGE International Examinations

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Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

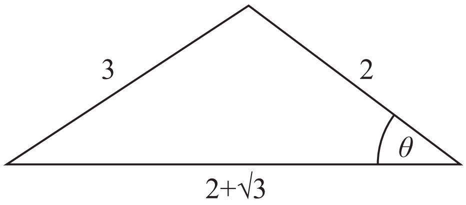
$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} bc \sin A$$

1 Solve the equation $|4x - 5| = 21$.

2 Given that the straight line $y = 3x + c$ is a tangent to the curve $y = x^2 + 9x + k$, express k in terms of c. [4]

3



Without using a calculator, find the value of $\cos\theta$, giving your answer in the form $\dfrac{a + b\sqrt{3}}{c}$, where a, b and c are integers. [5]

4 **(i)** Given that $y = \dfrac{1}{x^2 + 3}$, show that $\dfrac{dy}{dx} = \dfrac{kx}{(x^2 + 3)^2}$, where k is a constant to be found.

(ii) Hence find $\displaystyle\int \frac{6x}{(x^2 + 3)^2}\, dx$ and evaluate $\displaystyle\int_1^3 \frac{6x}{(x^2 + 3)^2}\, dx$. [3]

5 **(a)** The functions f and g are defined, for $x \in \mathbb{R}$, by

$$f : x \mapsto 2x + 3,$$
$$g : x \mapsto x^2 - 1.$$

Find fg(4). [2]

(b) The functions h and k are defined, for $x > 0$, by

$$h : x \mapsto x + 4,$$
$$k : x \mapsto \sqrt{x}\ .$$

Express each of the following in terms of h and k.

(i) $x \mapsto \sqrt{x + 4}$ [1]

(ii) $x \mapsto x + 8$ [1]

(iii) $x \mapsto x^2 - 4$ [2]

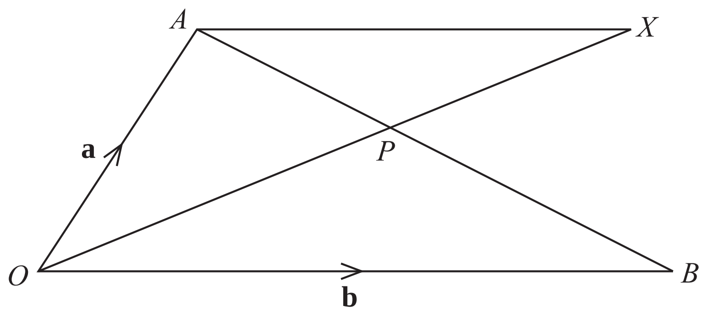
6 **Solutions to this question by accurate drawing will not be accepted.**

The points $A(1, 4)$, $B(3, 8)$, $C(13, 13)$ and D are the vertices of a trapezium in which AB is parallel to DC and angle BAD is 90°. Find the coordinates of D. [6]

7 **(a)** Given that $\tan x = p$, find an expression, in terms of p, for $\operatorname{cosec}^2 x$.

(b) Prove that $(1 + \sec\theta)(1 - \cos\theta) = \sin\theta\tan\theta$. [4]

8



In the diagram $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{AP} = \frac{2}{5}\overrightarrow{AB}$.

(i) Given that $\overrightarrow{OX} = \mu\overrightarrow{OP}$, where μ is a constant, express $\overrightarrow{OX}$ in terms of μ, $\mathbf{a}$ and $\mathbf{b}$. [3]

(ii) Given also that $\overrightarrow{AX} = \lambda\overrightarrow{OB}$, where λ is a constant, use a vector method to find the value of μ and of λ. [5]

9 The table shows experimental values of two variables x and y.

x	1	2	3	4	5
y	3.40	2.92	2.93	3.10	3.34

It is known that x and y are related by the equation $y = \frac{a}{\sqrt{x}} + bx$, where a and b are constants.

(i) Complete the following table.

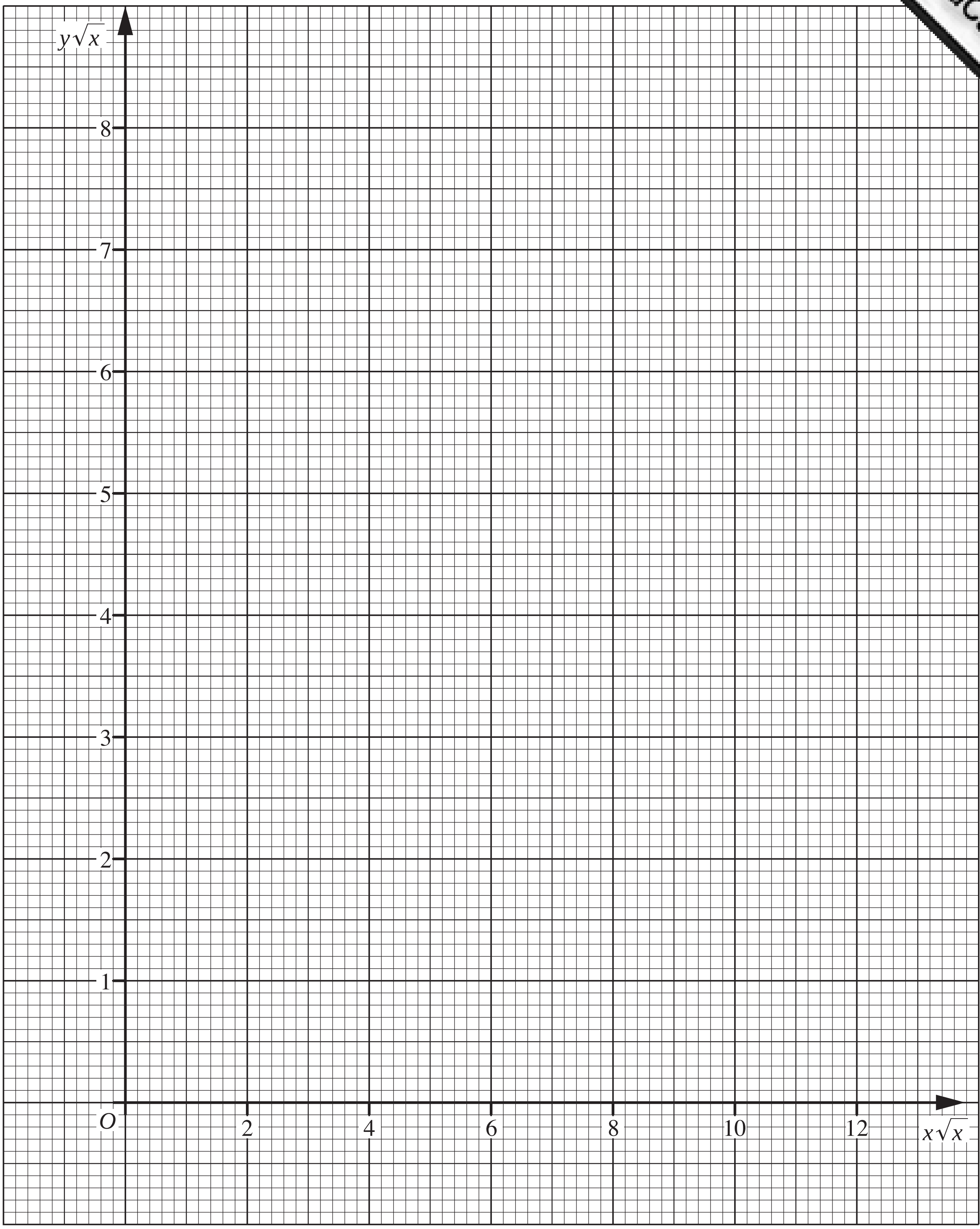
$x\sqrt{x}$					
$y\sqrt{x}$					

[1]

(ii) On the grid on page 11 plot $y\sqrt{x}$ against $x\sqrt{x}$ and draw a straight line graph. [2]

(iii) Use your graph to estimate the value of a and of b. [3]

(iv) Estimate the value of y when x is 1.5. [1]



y√x
8
7
6
5
4
3
2
1
O
2
4
6
8
10
12
x√x

10 It is given that $\mathbf{A} = \begin{pmatrix} 3 & 2 \\ 1 & -5 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix}$.

(i) Find $2\mathbf{A} - \mathbf{B}$. [2]

(ii) Find $\mathbf{BA}$. [2]

(iii) Find the inverse matrix, $\mathbf{A}^{-1}$.

(iv) Use your answer to part **(iii)** to solve the simultaneous equations

$$3x + 2y = 23,$$
$$x - 5y = 19.$$

[2]

11 **(a)** **(i)** Solve $\frac{5^{2x+3}}{25^{2x}} = \frac{25^{2-x}}{125^{x}}$.

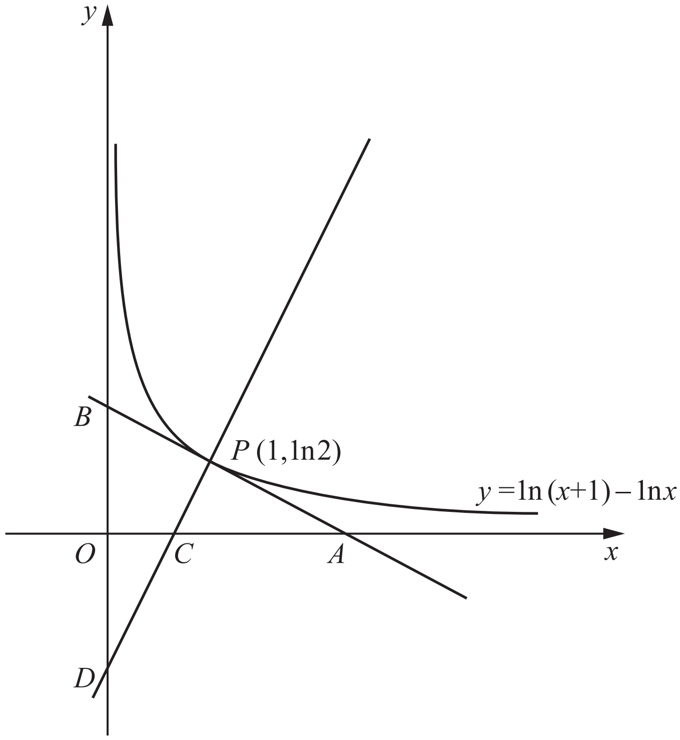
(ii) Solve 1g y + 1g(y – 15) = 2. [4]

(b) Without using a calculator, and showing each stage of your working, find the value o

$2\log_{12}4 - \frac{1}{2}\log_{12}81 + 4\log_{12}3$. [3]

12 Answer only **one** of the following two alternatives.

EITHER



The diagram shows part of the curve $y = \ln(x+1) - \ln x$. The tangent to the curve at the point P (1, ln 2) meets the x-axis at A and the y-axis at B. The normal to the curve at P meets the x-axis at C and the y-axis at D.

(i) Find, in terms of ln 2, the coordinates of A, B, C and D. [8]

(ii) Given that $\dfrac{\text{Area of triangle } BPD}{\text{Area of triangle } APC} = \dfrac{1}{k}$, express k in terms of ln 2. [3]

OR

A curve has equation $y = xe^x$. The curve has a stationary point at P.

(i) Find, in terms of e, the coordinates of P and determine the nature of this stationary point. [5]

The normal to the curve at the point Q (1, e) meets the x-axis at R and the y-axis at S.

(ii) Find, in terms of e, the area of triangle ORS, where O is the origin. [6]

Start your answer to Question 12 here.

Indicate which question you are answering.

EITHER	
OR	

Continue your answer here if necessary.

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